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An Exploratory Study of Interactivity in Visualization Tools: 'Flow' of Interaction

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This paper deals with the design of interactivity in visualization tools. There are several factors that can be used to guide the analysis and design of the interactivity of these tools. One such factor is flow, which is concerned with the duration of interaction with visual representations of information—interaction being the *actions* performed by users on these representations and the *reactions* given back by the representations. Four forms of flow can be identified: 1) continuous action, continuous reaction; 2) continuous action, discrete reaction; 3) discrete action, continuous reaction; and 4) discrete action, discrete reaction. Structuring micro-level interactions of tools based on these forms of flow can have varying effects on the cognitive processes of users. Based on this classification of flow, four versions of an interactive visualization tool were created and studied. The testbed for this study was 3D geometric solids—more specifically, Platonic and Archimedean solids. A multi-method empirical study was conducted to evaluate the usability of these four interfaces and their effect on learning, visual thinking, and exploration. This paper reports some findings of this study.

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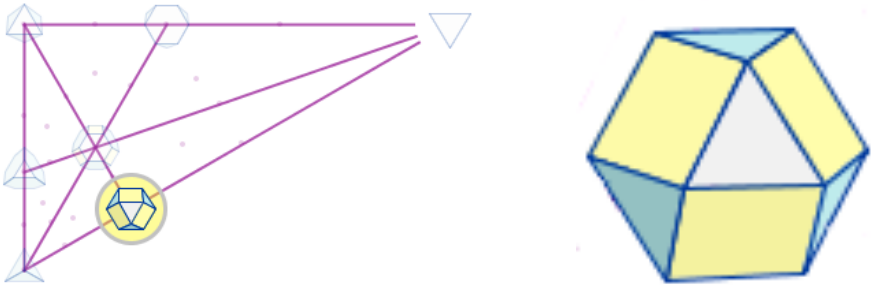
1. Introduction

Interface design is challenging. A substantial amount of research effort has been channeled into making tools easy and intuitive to use so that users can perform their tasks efficiently and effectively (Preece et al., 2002). These productivity tools are intended to minimize effort, while maximizing performance. In contrast, most visualization tools intended to facilitate knowledge-oriented activities have different goals. Instead of minimizing effort, they attempt to encourage and engage users into thinking and reflecting on the concepts being explored (Sedig et al., 2001). The interface of such tools, besides needing to be easy to learn and intuitive to work with, should be able to engage users in sense-making and knowledge construction.

Design of interactive visualization tools involves at least two related aspects: representation design and interaction design (Thomas & Cook, 2005; Spence, 2007). Representation design is concerned with the creation of appropriate visual encodings to support users' sense-making and other knowledge-oriented activities. The same information (e.g., concept, structure, and/or idea) can be depicted using different types of representations. Each type can influence how users reason about and think with the represented information. More importantly, the type of representation affects the amount of cognitive effort needed during interpretative analysis, reasoning, and mental manipulation (Larkin & Simon, 1987; Zhang & Norman, 1995; Cheng et al., 2001). For example, Figure 1 shows two types of representations. Figure 1a displays information using a map, which emphasizes the relational properties among the encoded 3D shapes. In contrast, Figure 1b presents only one object which can be changed into any of the shapes displayed on the map. Although all the shapes are present in latent form, users can view only one shape at the time. As a result, the map may be better for promoting the examination of relationships and comparative reasoning, while the single object representation may be more fitting for encouraging the investigation of structural properties (Sedig et al., 2005).



(a) (b)

Figure 1. Two types of representations of the same content

Interaction design, the second design aspect, is concerned with the manner in which users are allowed to operate or act upon the representations. Just as there can be different ways of representing information, there can also be varied ways by which users can interact with these representations (Sedig & Sumner, 2006). Some ways are more conducive for reflective thinking and learning purposes, while others can have unwanted learning outcomes (Svendsen, 1991; Golightly, 1996; Holst, 1996; Sedig et al., 2001).

For example, Svendsen (1991) reports that, in solving graphical puzzles, participants who used a command- or text-based interface reflected more and were more strategic in their actions, and, thus, learned more than those who used a mouse-based interface. Likewise, Sedig et al. (2001) describe how different operationalizations of mouse-based interactions affect children's processing of information and their degree of reflection on 2D transformational geometry concepts. Rather than allowing children to directly manipulate and move 2D geometric pieces (e.g., squares, triangles) to solve puzzles, they have to do so by acting upon the pieces indirectly, by manipulating another intermediary representation. The latter operationalization of the same interaction seems to have encouraged children to reflect more on their actions, leading to an improved learning experience.

Users' visual exploration and reasoning can be characterized at multiple levels of granularity (Gotz & Zhou, 2008). These levels are tasks, sub-tasks, actions, and events. Tasks and sub-tasks represent high-level structures of a user's cognitive processes, such as their goals and sub-goals. Tasks include scenarios such as analyzing the stock market or making sense of a particular shape transforming into another. Sub-tasks aid in the completion of tasks, and examples include finding the best and worst stocks, or finding how many edges a shape has. Actions refer to executable steps taken by users while completing a task, such as probing, arranging, composing, comparing, and so on (Sedig & Sumner, 2006). Events are at the lowest level of granularity. They are micro-level occurrences such as mouse clicks or drags, keyboard strokes, and menu-item selections, which are performed in unison with actions to complete tasks.

To perform tasks, one often needs to process information retrieved from internal representations, as well as information perceived from external representations, in an integrative and dynamic manner (Zhang, 2000). The theory of distributed cognition proposes the idea that cognition is not solely a product of one's internal cognitive processes, but rather an emergent property of interactions among internal and external representations and artifacts (Hollan, Hutchins, & Kirsh, 2000; Karasavvidis, 2002; Zhang & Norman, 1994; Salomon, 1993; Hutchins, 1995). In this context, external representations and artifacts are not only peripheral aids to cognition, rather they are essential components of cognitive processes, distributed across the mind of the user and their environment. External representations serve to anchor and structure cognitive behavior, by constraining, supporting, guiding, transforming, and enhancing the cognitive processes and activities of users (Zhang, 1991; Sedig & Liang, 2008). Interactive visualization tools comprise a subset of these external representations and artifacts. These tools are situated at the level of the interface, playing a mediating role between users and the information with which they are interacting (Sedig & Liang, 2006).

It is often the case that researchers discuss only high-level details when designing visual tools for the purpose of supporting knowledge-oriented activities. This study, however, demonstrates that the choice of actions, coupled with low-level events, has an effect on the cognitive processes of users. It is the structure of interaction, that is, the micro-level interactions in the form of events, which can be operationalized in certain ways in order to affect the reasoning and sensemaking processes of users.

In the context of interaction design, the facet that examines how interactions can and should be operationalized is interactivity design. Recently, Sedig and Liang (2006) have devised a framework that can be used to analyze and design the interactivity of

visualization tools. This paper extends that previous research and explores an application of the factor of interaction flow and its effect on visual exploration and reasoning. To this end, an educational visualization tool was created and an empirical usability study was conducted with four versions of the tool involving high school students. This paper reports some findings of this study.

The remainder of this paper is structured as follows. The next section will provide some terminological and conceptual background information, including an elaboration of the concepts of interaction, interactivity, and the factor of flow. Next, the research methodology will be described, including the visualization context, the study tool, and study design. The section after that will report the results of the study, followed by a discussion section. Finally, the last section will present the conclusions derived from this study.

2. Background and terminology

Interaction is concerned with the dialectic process of a user acting upon a representation through the intermediary of a human-computer interface, and the representation responding back to this action for the user to interpret. Interaction can act as an epistemic extension of static representations by adding a temporal dimension to them. This temporality of representations can extend their communicative power as users can dynamically explore them in order to discover their latent meanings and hidden properties. Interaction enables users to transform, manipulate, and navigate through the distinct elements and features of representations. As such, interaction has a great influence on *what* and *how* a user explores, reasons with, and learns from the representations. That is, the manner in which interaction is operationalized can determine whether and how different cognitive processes are supported, enhanced, canalized, guided, promoted, scaffolded, and/or constrained (Sedig & Liang, 2006). In this context, interaction design can play an important role in visualization tools, as designers can operationalize an interaction in such a way that promotes thinking and reflection of the represented content. More importantly, this can be achieved in a tacit manner, just as users explore the embedded representations in the tool.

Interaction and interactivity, although closely related, are distinct concepts. Whereas interaction refers to the action and response provided in return, interactivity refers to the *feel*, *properties*, and *quality* of interaction (Sedig & Liang, 2006). For instance, interaction with a representation can be through manipulation. However, this interaction can be through direct manipulation of a representation or through indirect manipulation—i.e., via a second intermediary representation. Whereas in both cases interaction is manipulation, the feel and properties (i.e., interactivity) of direct and indirect manipulation are different, which can affect users' attentive processes and degree of learning (for a more detailed discussion and examples, see Golightly, 1996; Sedig et al., 2001). Interactivity of representations (or tools) can affect the amount of cognitive effort users put into making sense of the inherent features and meanings of the representations, how their different thought and reasoning processes are supported, how deeply they learn, how engaged they become with the tool, and the interchange between them and the tool (Kirsh, 1997; Burgoon et al., 2000; Preece et al., 2002).

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Sedig & Liang (2006) have suggested that a set of factors can assist in the analysis and design of the interactivity of visualization tools. These factors can serve as a descriptive and conceptual framework to aid in the design of such tools. Each of these factors of interactivity can be operationalized and structured in certain ways, which can affect the cognitive processes of users. One of these factors, which has been chosen as the focus of this study, is flow[1]. In this study the low-level operational structure of interaction has been analyzed and tested, and has been shown to have an impact on the way users reason and make sense of represented information. The study was performed to validate one of the elements of the previous framework (flow) and to examine its impact on users' cognitive processes.

Flow is a factor that is present in all types of interaction. It is concerned with the duration of interaction with a representation. As such, flow is equally about the action performed on the representation, as well as the reaction given by the representation. In the context of interaction with information spaces, many researchers (such as Spence, 2007) refer only to continuous action, or to continuous action coupled with continuous reaction, which they then call continuous interaction. That is, no distinction is made between the continuity or discreteness of the action and that of the reaction. This approach may be fine for certain purposes, but interaction should be examined in greater detail for the purpose of facilitating knowledge-oriented activities.

Both action (cause) and reaction (effect) aspects of interaction occur in time. This being the case, interaction flow can happen in two ways: continuous and discrete. Whereas continuous flow occurs when both cause and/or effect happen in a fluid, uninterrupted manner over a span of time, discrete flow occurs when both cause and/or effect take place in a distinct, separate manner, at one instance in time. Given continuous and discrete flow, users can perceive interaction in four ways: continuous action, continuous reaction (C-C); continuous action, discrete reaction (C-D); discrete action, continuous reaction (D-C); and discrete action, discrete reaction (D-D). Using the figures below, instances of applying the four forms of flow are provided next. In the event of a user wanting to interact with a representation, the forms can guide the operationalization of action and reaction. An instance of C-C flow can be seen in Figure 2. This data analysis tool (www.miner3d.com) is meant to assist users in understanding the effects of drugs in an Alzheimer study. Figure 2a shows the original window with multiple sliders on the right that are controlled by continuously dragging the bidirectional arrows (i.e., continuous action). The effect of dragging the arrows is shown in Figure 2b, and is simultaneously and continuously displayed in a fluid manner and without any time delay (i.e., continuous reaction). Figure 3 shows another example of C-C flow, this time in a tool for exploring planets (Google Earth). In order to rotate the earth, users click and drag in the direction of rotation (i.e., continuous action), and the effect is shown immediately and fluidly (i.e., continuous reaction).

[1] Other factors include: affordance, cognitive offloading, constraints, distance, flexibility, focus, and transition (Sedig & Liang, 2006).

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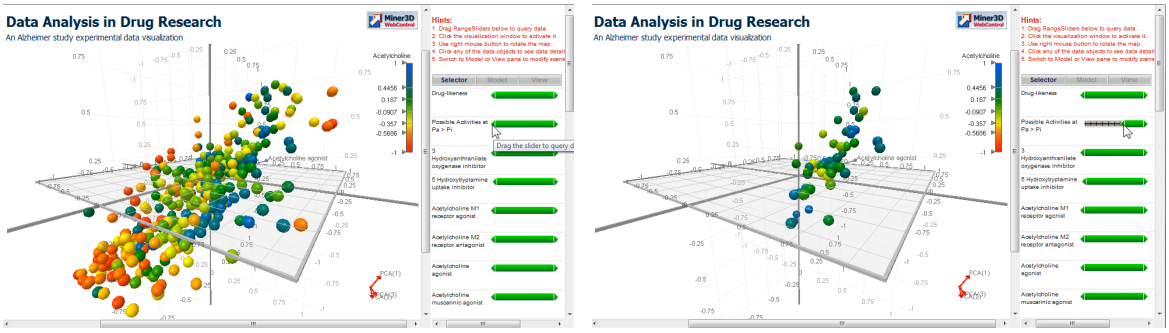
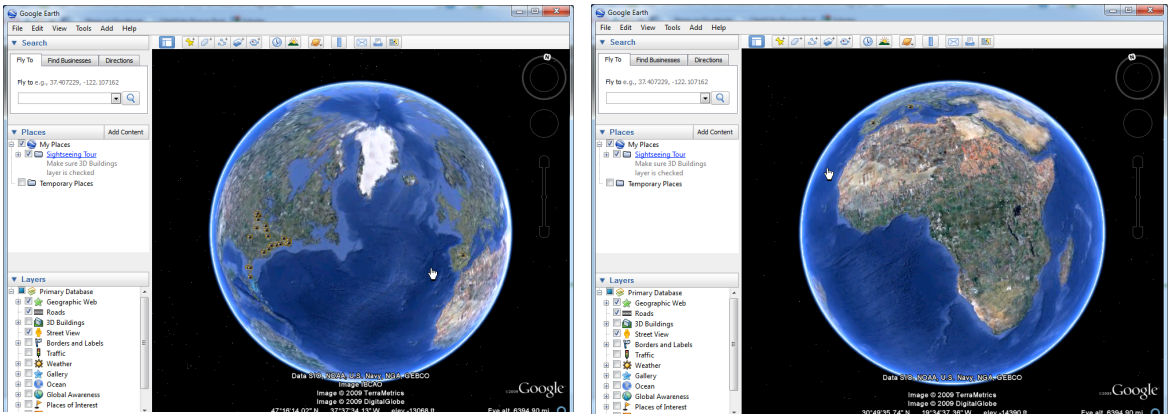


(a) (b)

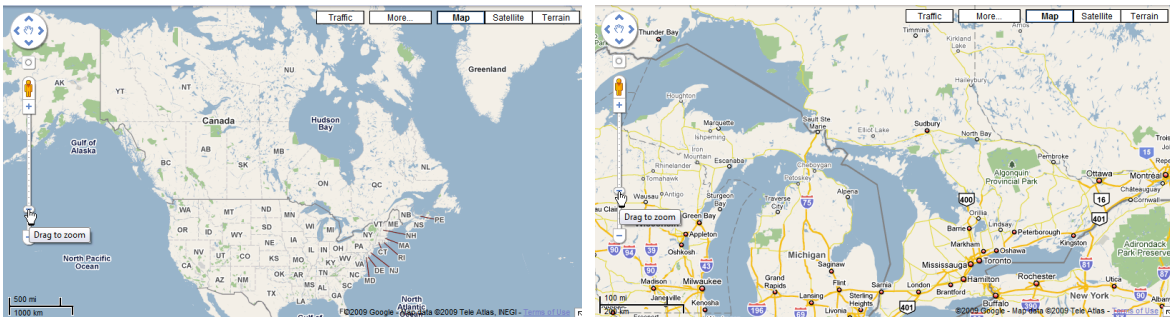
Figure 2. C-C flow with visual analytics software
Used with permission from www.miner3d.com



(a) (b)

Figure 3. C-C flow with Google Earth (www.earth.google.com)

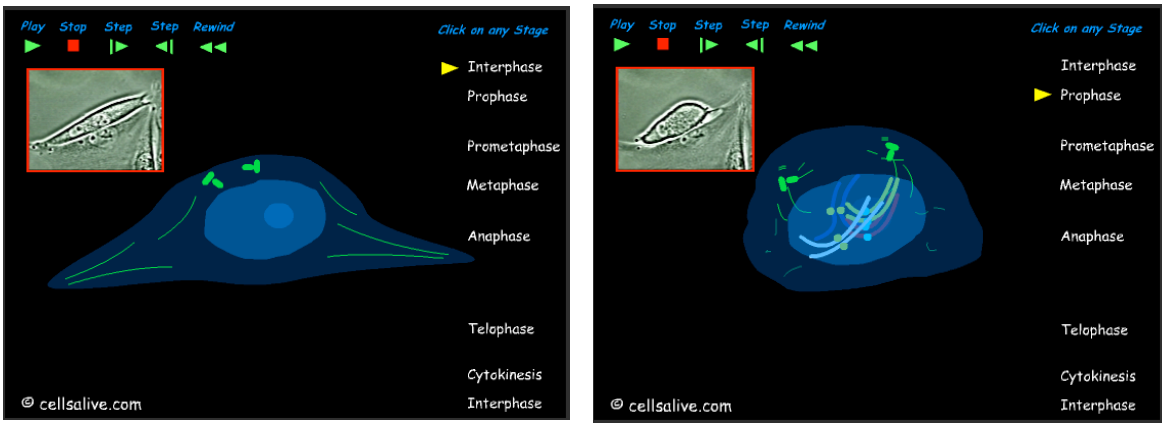
In the case of C-D, the action is performed in the same manner as in C-C—i.e., continuously. However, the effect of the action is not continuously shown, but displayed only after the action has been fully completed (i.e., discrete reaction). Figure 4 shows an example of C-D flow in a tool for geographic analysis and route-panning (Google Maps). Users drag the slider on the left (Figure 4a) to increase or decrease the zoom in the same manner as the C-C examples above, however the reaction does not occur until the dragging has stopped and the mouse button has been released (Figure 4b).



(a) (b)

Figure 4. C-D flow with Google Maps (www.maps.google.com)

In the case of D-C, instead of allowing dragging of components (i.e., continuous action), these can only be clicked on (i.e., discrete action). This type of flow is also available with Google Maps, when using the pan tool in the top left corner. When a mouse-click takes place to pan in a particular direction (i.e., discrete action), an animated process shows the effect of the action smoothly and continuously (i.e., continuous reaction).



(a) (b)

Figure 5. D-D flow with a biology tool
Used with permission from www.cellsalive.com

Finally, in the case of D-D, similar to D-C, the action is performed through mouse-clicks. When a mouse-click takes place (i.e., discrete action), the effect is shown instantly (i.e., discrete reaction). Figure 5 provides an example of a tool for learning about cell mitosis that uses D-D flow. Users are initially presented with the first stage of mitosis (Figure 5a). By clicking on any of the stages on the right side of the tool (i.e., discrete action), the effect will be shown immediately, with no transitional process being shown (i.e., discrete reaction). Figure 5b shows the result of a user clicking on the second phase. Although there might be other ways of operationalizing these four forms of flow, the important point to note here is that each form can lead to different styles of exploration. This might even affect how one reasons with the representations and the amount of effort needed to understand the transitional changes of the representations.

The categorization based on the concept of flow is useful for designers of visualization tools as it can assist them in deciding how to operationalize interactions in ways that may encourage greater reflection and exertion of mental effort. For example, designers can control whether actions are performed in a continuous or discrete fashion. Similarly, designers can choose whether the reactions to these actions are provided continuously or discretely. Norman (1988) has devised an action cycle framework that describes the steps taken to achieve a goal while interacting with an external artifact. Users must devise a goal or intended task, and decide on actions needed to meet the desired goal. Once this is done, users execute the actions, using low-level events in order

to meet their goal. Now that an action has been completed, users must make sense of the reaction given from the tool, and compare the result to the original goal. Norman refers to the amount of effort needed to evaluate the effect of the interaction with the tool and how well original goals have been met as the gulf of evaluation. When flow is in a continuous manner, it is expected that the gulf of evaluation would be small. But when the flow of an interaction is discrete, the gulf of evaluation may be bigger, since more mental processing is required to make sense of the changes. Continuity of action might be more natural and provide users with a greater sense of control over their exploration. However, as reversal of action is easy, when provided with continuous actions, users' exploration might be less planned and more opportunistic. When actions can only be performed discretely, reversal of actions might be more difficult, encouraging users to do more planning and thinking before committing to an action. In the same way, there are differences between reactions that are provided continuously or discretely. An instance of how continuous reaction can be provided is animation. Animation might be beneficial for understanding as it can draw users' attention to what is important. One of the drawbacks of animation, however, is that it may increase the visual explicitness of representations, which might then cause the reduction of mental effort and reasoning on the part of users due to overconfidence in the amount of knowledge obtained from the animation (Jones & Scaife, 2000). In contrast, when reactions are provided discretely, users only see the initial and final resulting states of the representation. This being the case, users need to connect the two by mentally visualizing the intermediate stage(s). This can be difficult for some users, especially when the initial and final states of the representations are conceptually distant from one another. As such, by creating this disconnect, discreteness of reactions can be used to promote deeper reasoning during exploration.

Similarly, designers can manipulate how cause and effect are provided during user-representation interaction. Generally, in tools that facilitate knowledge-oriented activities, cause and effect are linked; however, they can be separated. As it allows users to perceive cause and effect concurrently, linking the two in time may be more intuitive and natural, but it might require less effort and reflection during interaction. On the other hand, separating cause from effect (e.g., when the provision of effect is withheld and delayed) may implicitly encourage users to visualize or conceive the effect in their minds first. When the effect is actually given, users can confirm or rectify their earlier mental conceptions. As such, this delay might help elicit greater reflective, self-monitoring, and self-correcting cognition (Corbett & Anderson, 2001), as opposed to a more automatic, shallow processing of information promoted when cause and effect are provided without such delay (Liang & Sedig, 2007).

The following section presents the research methodology, which includes the visualization tool, the context domain, the four different interfaces of the tool, and the usability evaluation.

3. Research methodology

A visualization tool was created to investigate the effect on visual exploration and reasoning when interaction is operationalized based on the four forms of flow. This tool allowed its users to interact with representations of 3D geometric solids and explore their structural properties and relational transformations. Four versions of the tool were made

available, with each version corresponding to an application of one of the four forms of interaction flow. A usability study was conducted involving high-school students.

3.1.1. Problem domain and justification

The problem domain chosen to conduct this research is 3D geometry, and, in particular, Platonic and Archimedean solids. Platonic solids are 3D shapes that are composed of one type of regular polygons[2]. Because of this property, Platonic solids look identical from every vertex. A cube, for example, is a Platonic solid since it is composed of only squares, with three squares meeting at each vertex. Archimedean solids, on the other hand, are shapes composed of two or more types of regular polygons. Platonic and Archimedean solids are closely related and can be obtained from one another. Simultaneous truncation of all the vertices and/or edges of a Platonic solid can result in other Platonic or Archimedean solids. For example, the truncation of all eight vertices of the cube will result in the truncated cube, which is an Archimedean solid as it is composed of two regular polygons (i.e., octagons and triangles). This truncation process is similar to that of cutting or chopping off successively and symmetrically the vertices and/or edges until the faces of the solids become regular (see figures 6 and 7).

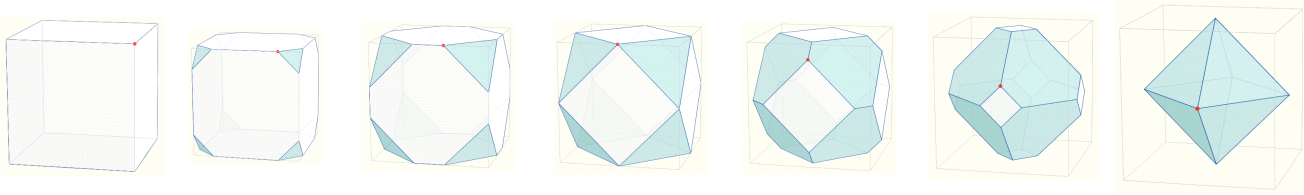


Figure 6. Sequence of snapshots in the truncation of all the *vertices* of a cube

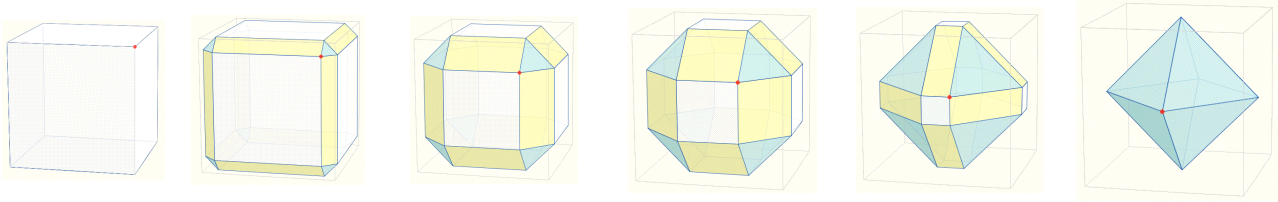


Figure 7. Sequence of snapshots in the truncation of all the *vertices* and *edges* of a cube

Platonic and Archimedean solids provide a viable testbed for this study. This domain area was chosen for the following three reasons. First, although Platonic and Archimedean solids can be described using algebraic (i.e., symbolic) representations, they readily lend themselves to be communicated using visual (i.e., diagrammatic) representations. Because the emphasis of this research is on visual reasoning and exploration, the solids can serve well for examining whether design based on the flow factor can have an effect and what this effect might be. Second, the successive truncation of vertices and/or edges represents transitional, metamorphic processes (Morey et al., 2001). Understanding these processes might not be easy and requires thinking and

[2] A regular polygon is a 2D shape whose sides are all of equal length. Hence, a rectangle is not a regular polygon, but a square is a regular polygon.

reasoning about how one solid changes or morphs into another (see figures 6 and 7). That is, users need to compare two solids (i.e., a beginning morphing solid and a final morphed solid) and the many intermediate solids in between. It is possible to manipulate how users act upon the solid representations and how the representations respond to users' actions according to the four forms of flow. Third, transformation of 3D geometric solids has parallel and widespread applications in other domain areas, such as geological, biological, and chemical transitions. For example, various metamorphic rock types can form from sedimentary rock through changes in temperature, pressure, deviatory stress (tension, compression, shearing), or chemical active fluids. As such, results from this study can have implications in the design of visualization tools intended to support learning in other domain areas.

To conduct this study, four versions of a visualization tool dealing with Platonic and Archimedean solids were created. Each version corresponded to an implementation of one form of interaction flow. Before describing how users can interact with the different versions of the tool, its interface layout and main components will be briefly provided next, as they are similar in all four versions.

3.1.2. Study tool: Interface layout and components

Figure 8 shows a screen capture of the layout and components of the study tool. The tool has four panels: three 'map' panels (panels 1-3) and the Enlarged Solid Panel (panel 4). The three maps group related solids and lay them out as triangular maps. The maps communicate the transitional processes by which solids can be obtained from each other. Each map corresponds to a Platonic solid. The solid displayed on the top right corner represents the base Platonic solid, whereas the remaining solids (displayed on the intersecting lines) indicate the locations of other regular solids that can be obtained from the base Platonic solid. Three types of spatial information are encoded in these maps: 1) landmark, or the location of each regular solid on the maps; 2) route, or the transitional routes connecting the solid landmarks; and 3) survey, the entire space of landmarks and transitions in the maps. As such, the manner in which the maps organize the space of solids is intended to help users in understanding their relationships and transformations (Glasgow et al., 1995; Tversky, 2000).

The maps display three types of solids: *Selected*, *Twin*, and *Regular*. The Selected Solid represents the currently in-focus shape on the maps. There is at most one Selected Solid and the map where it is located becomes the only active map. This solid is emphasized by a surrounding yellow circle. For example, the current Selected Solid is shown in Figure 8 and is located on the Cube Map (Panel 3). Users can move the Selected Solid anywhere on the active map (how users move the Selected Solid differs according to each version and is discussed later). A Twin Solid has the same structural properties as the Selected Solid, but differs in the way it is obtained. This is because Platonic solids share similar properties and are closely related. For instance, in Figure 8, the Selected Solid (Panel 3) is on the Cube Map but there is also one Twin Solid on the Octa Map (Panel 2). This means that the same solid can be obtained from either the octahedron or the cube. Also, besides showing structurally-congruent shapes, the Twin Solid enables users to navigate across maps. That is, users can only move to another map when there is a Twin Solid on it. This is a design constraint put in place to encourage users to explore and understand the relationships among the solids.

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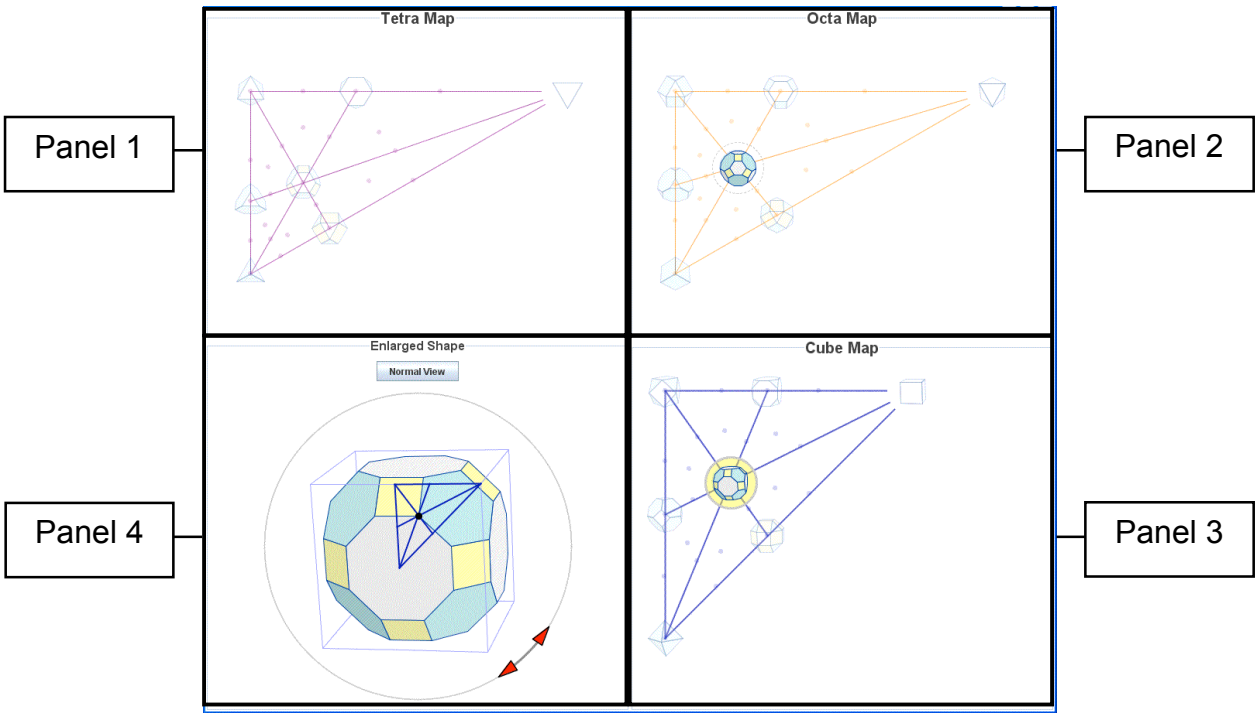
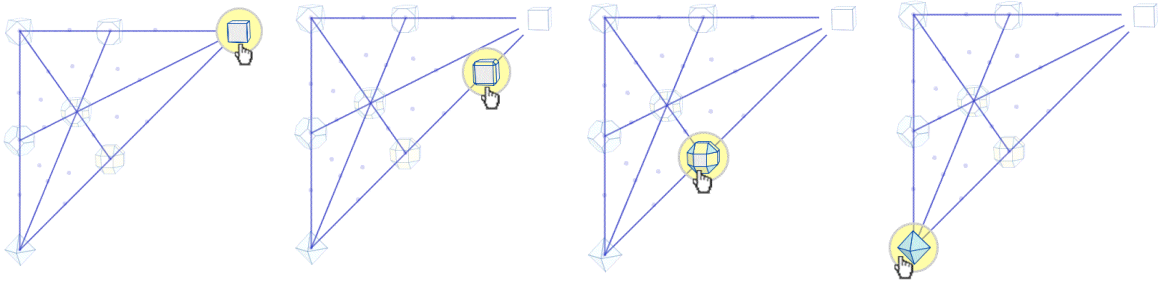


Figure 8. Screen capture of the visualization tool

The fourth panel displays the enlarged version of the Selected Solid. It is provided to assist users in exploring the structural properties of the solids with more ease. The Enlarged Solid displays three kinds of information: 1) the deriving Platonic solid, represented by the outer wire frame; 2) the location of the Selected Solid, indicated by the dot located on the triangle; and 3) the actual in-focus solid, displayed as the internal shape with colored faces. The Enlarged Solid and the maps are dynamically-linked (see Figure 9). Dynamic linking is important for learning systems where multiple representations are used (Scaife & Rogers, 2005; Sedig & Liang, 2006). In the tool, once users have identified a solid they want to explore further by interacting with the Selected/Twin Solid, they can switch their interaction and focal to the Enlarged Solid, where they can rotate the solid to perform exploratory and other related activities.



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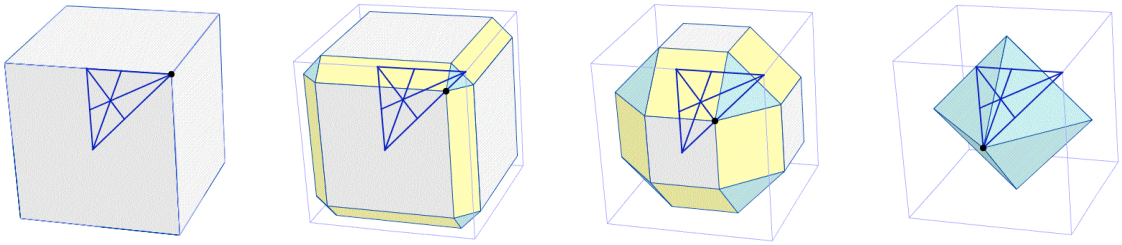


Figure 9. Snapshots of the dynamic linking of the Cube Map and the Enlarged Solid

Before describing in detail, Table 1 shows the summary of the four different versions of the study tool based on how actions can be carried out and how the effects are displayed. When interacting with the Selected/Twin Solid, both the Selected and Enlarged solids are affected. On the other hand, when interacting with the Enlarged Solid, the effect is restricted only to itself—i.e., the Selected/Twin Solid is not affected by it.

		C-C	C-D	D-C	D-D
Maps (Morphing of both the Selected/Twin Solid and Enlarged Solid)	Action	Continuously dragging of Selected/Twin Solid	Continuously dragging of Selected/Twin Solid	Discretely clicking on locations within the maps	Discretely clicking on locations within the maps
	Reaction	Continuous update of the Selected/Twin Solid	Continuous update of the Selected/Twin Solid	Continuous and automated update in the form of animated movement and morphing of the Selected/Twin solids	Instant jumping of the Selected/Twin solids from current location to new location
		Continuous update of the Enlarged Solid to match the Selected Solid	Instant update of Enlarged Solid occurs only after mouse drag is stopped	Continuous and automated update in the form of animated morphing of the Enlarged Solid	Instant update of the Enlarged Solid to match the Selected Solid
Enlarged Solid (Rotating of the solid)	Action	Continuously dragging of the Enlarged Solid	Continuously dragging of *outer frame* of Enlarged Solid	Discretely clicking on the arrows located around the Enlarged Solid	Discretely clicking on the arrows located around the Selected Solid
	Reaction	Continuous update of the Enlarged Solid	Instant update of *internal solid* matching it to the *outer frame* occurs only after mouse drag is stopped	Continuous and automated update in the form of animated movement of Enlarged Solid	Instant update of the Enlarged Solid rotated towards the selected rotational direction

Table 1. Different implementations of the study tool based on the four types of interaction flow

3.1.3. Continuous-Continuous (C-C) interface

The C-C interface is shown in Figure 8. In this interface, actions can be performed continuously, with reactions provided in a continuous manner. As stated earlier, users can interact with the active map, indicated by its darker colored lines and the presence of the Selected Solid. In this interface, users can position the mouse-cursor on the Selected Solid and, while pressing the mouse-button, drag the solid around continuously and position it anywhere on the map (i.e., continuous action). As the Selected Solid is being dragged, the Enlarged Solid changes and updates itself continuously, matching shape of Selected Solid (i.e., continuous reaction). Figure 9 shows an instance of C-C interaction.

Similarly, when interacting with the Enlarged Solid, both action and reaction aspects are continuous. For instance, when users want to rotate the solid, they can do so by

positioning the mouse-cursor on the solid and dragging it towards the direction of their preference. This results in constant and continuous update of the Enlarged Solid.

3.1.4. Continuous-Discrete (C-D) interface

The C-D interface looks exactly like the C-C interface. Also, the way in which actions can be performed is similar to the C-C version—i.e., continuously. The difference between these two interfaces is primarily based on how reactions are provided. During continuous action with the Selected Solid, the Enlarged Solid does not update itself until the action has stopped, when the mouse-button is released—i.e., discrete reaction. That is, although the Selected Solid can be moved around the map continuously, only after this action is no longer performed does the Selected Solid instantly redisplay itself to match the Selected Solid—i.e., discrete action. The C-D form of interaction flow attempts to separate cause and effect. This particular application of C-D in the tool, by beholding and delaying the provision of reactive feedback, is intended to promote visualizing the resulting solids in users' minds without the constant need for visual feedback. The aim is to facilitate greater reflection and thinking and the development of retentive, self-monitoring, and self-correcting processes (Corbett & Anderson, 2001).

In the same way, when interacting with the Enlarged Solid, only the reaction aspect differs from the C-C interface. In the C-D interface, while acting continuously upon the solid, only the Platonic wire frame is continuously being updated, leaving the internal solid unchanged (see Figure 10). The continuous update of the wire frame is intended to provide some limited operational feedback so that users are cognizant that an action is being performed. The internal solid updates itself only when the mouse action has stopped, after which the solid is instantly changed to match the orientation of the wire frame. This delay is intended to encourage users to perform more premeditated planning of and thinking about the final orientation of the solid before actually releasing the mouse- button.

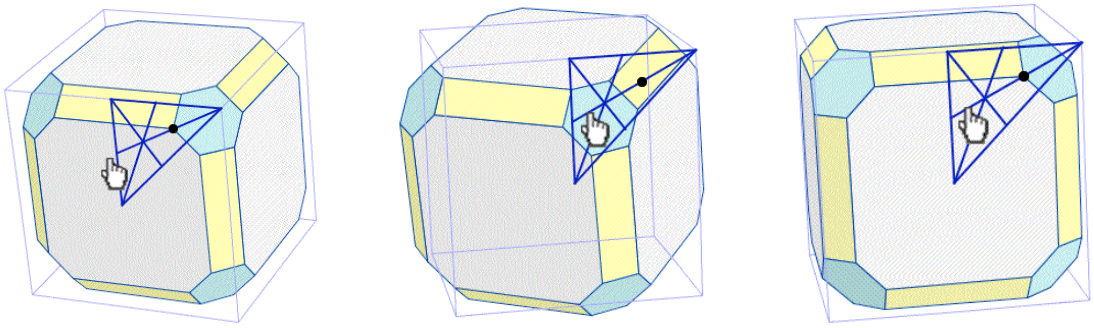


Figure 10. Snapshots of interacting with the Enlarged Solid based on the C-D form

3.1.5. Discrete-Continuous (D-C) interface

In the D-C interface, actions are to be performed in a discrete manner, while the reactions are presented in a continuous fashion. This interface restricts actions to be performed only through mouse-clicks. To perform an action, users can click anywhere on the active map. After each mouse-click, a smooth animation shows the Selected Solid moving towards the final destination—i.e., the location where the mouse-click has taken place. Similarly, while the Selected Solid is moving, the Enlarged Solid updates itself continuously to match the morphing Selected Solid. Figure 11 shows a sequence of

snapshots displaying the animated movement of the Selected Solid after an action has been issued, where the final destination of the animation is indicated by the hand-cursor. Discrete actions happen one at a time. Upon committing to an action, users lose control of communication until the animation ends, only then can they commit to the next action. As there is time cost associated with each action and reversal of action is not easy, users might need to resort in more premeditated actions.

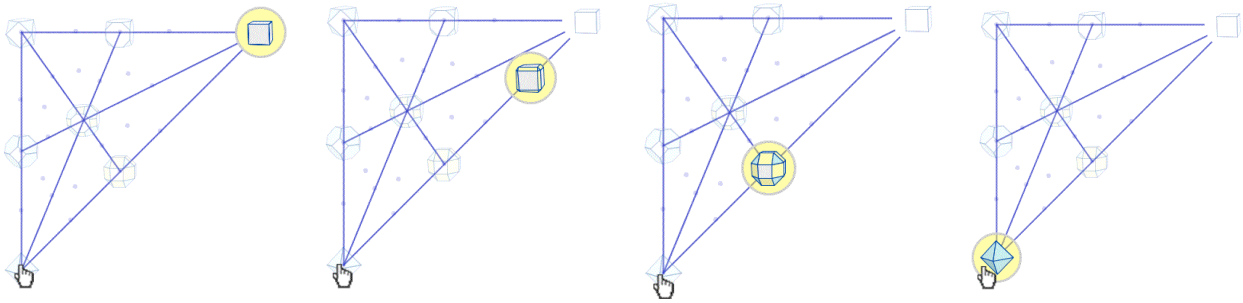


Figure 11. Snapshots of the animated movement of the Selected Solid

In the D-C interface, to allow discrete action with the Enlarged Solid, a set of interactive directional arrows have been added to the solid representation (see Figure 12). To act upon the Enlarged Solid, users first need to choose a direction towards which they want to rotate the solid. Then, to actually rotate the solid, users can click on the arrow matching the chosen direction. After the action is issued, a smooth animation is run showing the continuous rotation of the solid by a predefined amount.

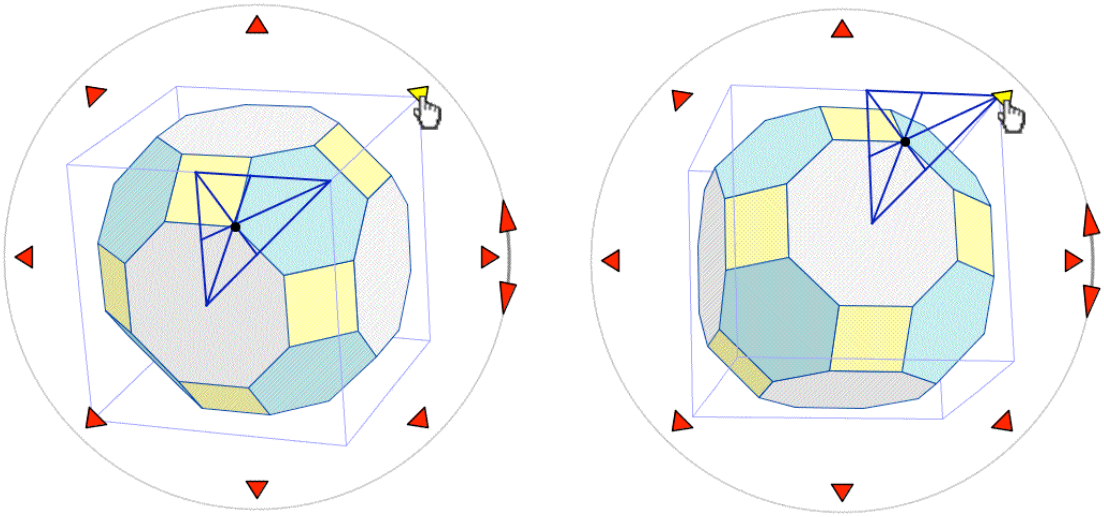


Figure 12. Interacting with the Enlarged Solid through the directional arrows

3.1.6. Discrete-Discrete (D-D) interface

The D-D interface is exactly the same as the D-C interface. Also, the way by which actions are performed are the same for both interfaces. The difference between them lies in the manner in which reactions are presented. While in the D-C interface reactions are continuously provided in the form of animations, in the D-D interface, reactions are discrete, occurring instantly after a mouse-click action. So, unlike the other interfaces, the

D-D interface allows 'jumping' from one location to another within the active map. This is the case since the Selected Solid can move from its current location to any location using mouse-clicks, without the need to observe the intermediate solids. To an extent, this is the fastest way for users to navigate from one location to another.

Similar to the D-C interface, interaction with the Enlarged Solid is through the directional arrows. The difference, however, is that instead of providing continuous reaction in the form of animation, in the D-D interface, the response is given as an immediate rotation of the solid in a chosen direction.

3.2. Usability evaluation

This section describes the empirical usability study that was conducted to evaluate the four interface versions of the visualization tool. The study was intended to investigate whether and how each operationalization of the four of forms interaction flow affects visual thinking, reasoning, and exploration.

3.2.1. Design

A multi-method (both quantitative and qualitative) research design was used, incorporating a number of types of data-gathering instruments (achievement results, video transcripts, video screen captures, interviews, and direct observations). The multi-method design was used to help triangulate and cross-validate the different types of findings and results. This study compared four groups, each group interacting with one of the four interfaces. As such, this study did not have a control group in the traditional sense. Rather, it was intended to offer a comparative evaluation of the four interfaces. This method of comparing different approaches without having a control group is common in educational settings (Ary et al., 2003).

3.2.2. Participants

Forty grade 11-12 high-school students, ages ranging from 16 to 19, participated in this study[3]. Teachers from a local school were initially approached to see whether they would like their students to participate in the study. After the teachers invited their students to participate, the first forty who responded were selected. None of the participants had used the visualization tool before and all students were computer-literate. Also, none of the participants had been taught the mathematical concepts embedded in the tool in formal educational settings.

3.2.3. Sources of data

Five sources of data were used to do the analysis and evaluation of the visualization tool: 1) achievement results, obtained from the pre- and post-test scores; 2) video and screen capture transcripts, obtained from video tapes and screen captures of the participants' interactions with the tool; 3) questionnaire answers, obtained from the survey conducted at the end of the study; 4) interview transcripts, obtained from video tapes of the post-hoc interviews with some participants; and 5) direct observations, obtained from the

[3] One participant could not complete the study; so, the sample size was reduced to thirty-nine.

recordings made of participants' overall patterns of usage and body language not easily captured through video recordings.

3.2.4. Procedure

All participants were asked to complete a multiple-choice geometry test before and after interacting with the tool. The pre- and post-test contained the same questions. After the pre-test, participants were randomly distributed into four groups (one group for each interface: C-C, C-D, D-C, and D-D), with each participant assigned to work with one other participant on a set of question-based tasks. Pairing of participants (with 5 pairs for each group) is based on the co-discovery usability testing method (Kennedy, 1989). In this method, two participants learn how to use and work with a program while conversing with each other. This conversation helps evaluators gain a better understanding of the usability of the program. In the study, each pair interacted with the program for about two and a half hours: three sessions of about 45 minutes on three consecutive days were held during normal school hours. As participants interacted with the visualization tool, they were videotaped and written notes were made of their overall pattern of use and their verbal comments. Also, a computer screen-capturing program recorded participants' interaction with the tool along with their verbal comments. Finally, after the last session with the tool, all participants wrote a post-test, which was identical with the pre-test. After the post-test, participants completed a questionnaire which collected feedback about their impressions of the tool. Some participants were also briefly interviewed to help clarify some of the comments they made during their interaction with the tool and the answers they gave on the tests and questionnaire.

3.2.5. Tasks

Participants were requested to complete sixteen question-based tasks using the visualization tool. These tasks were intended to provide participants with predefined goals to make collection of data efficient and effective within the limited duration of the study. The tasks were designed based on the types of potential actions users might perform to make sense of representations as discussed by different researchers (e.g., Keller & Keller, 1993; Zhou & Feiner, 1998). Some of these actions include: identify, locate, distinguish, categorize, compare, associate, correlate, and rank. Each task was composed of several questions. These questions have different levels of difficulty. Some questions required only one type of action; whereas others are more involved and required a combination of several actions. Examples of questions which required only one type of actions included: 'How many faces, corners, and edges does the above shape have?' requiring a 'identify' type of action; 'Can you get [picture of an Archimedean solid] by cutting off the corners of [picture of a Platonic solid]?' requiring a 'associate' type of action; or 'From which of the following three shapes [figures of the 3 Platonic solids] can you get this shape [figure of an Archimedean solid]?' requiring a 'correlate' type of action. Instances of more involved questions included: 'What type of faces do you get when you cut off the edges of any shape?' requiring both 'compare,' 'distinguish,' and 'categorize' types of actions; and 'What is the relationship between the shapes you get from the cube and the shapes you get from the octahedron?' also requiring 'compare,' 'distinguish,' and 'categorize' types of actions.

4. Results

This section reports the results of the study in two subsections: 1) analysis of the quantitative results, which include test achievement results, task-performance related data, and questionnaire answers; and 2) analysis of the qualitative results, which include comments made by participants about each version of the tool during their interaction with it and during interviews as well as notes taken by researchers.

4.1. Analysis of the quantitative results

This subsection is intended to assess whether interacting with the tool had any effect on the learning (based on the test results) and exploration of the 3D solids (based on the questionnaire answers and task-performance data).

4.1.1 Geometry Test achievement results

Generally, participants achieved a higher score in the post-test than on the pre-test. As Figure 13 shows, the media of the post-test is much higher than the media of the pre-test.

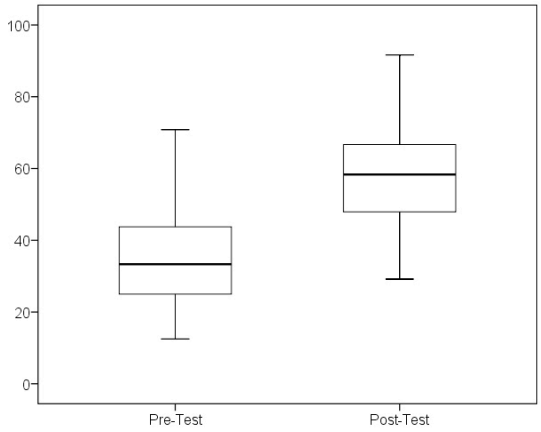


Figure 13. Boxplot of pre- and post-test results in % for all participants

Table 2 presents the descriptive statistical data for all participants. As expected, the mean for the post-test is higher than the pre-test. Table 3 shows the results of the overall statistical analysis. Regardless of the interface they used, the mean increase for all participants was 22.8% with a standard deviation of 12.9%. At an inference level, a one-sided paired-sample *t*-test was conducted. The hypothesis tested was that participants would perform better on the post-test. The results indicate a significant improvement in test scores ($t_{diff}(38)=10.97$; $P<.001$). Thus, overall, the performance of all participants improved after interacting with the tool.

	Mean	N	Std. Deviation	Std. Error Mean
Pre-Test	36.269	39	13.310	2.131
Post-Test	59.073	39	15.172	2.429

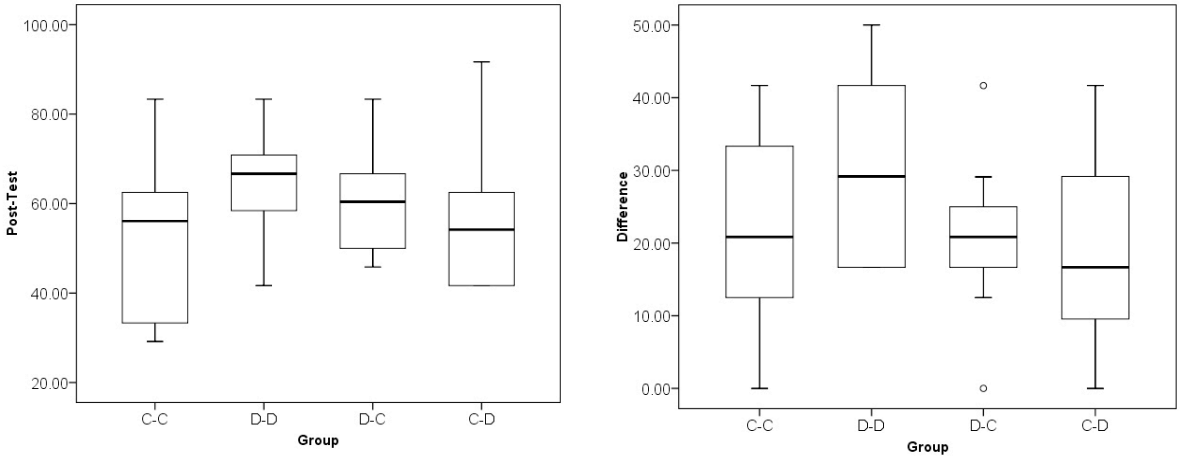
Table 2. Overall descriptive statistical data

Dependent *t*-test for overall results for all participants

	Mean difference between pre- and post-test results	Std. deviation	*t*-statistic	df	*p*-value
All participants	-22.804	12.986	-10.966	38	<.001

Table 3. *t*-test analysis of the difference in % between pre- and post-tests

The following analysis investigates how the individual four groups performed and whether there was any significant difference because of the version students used. The scores for all participants within each treatment group increased in the post-test. Figure 14a shows the post-test results organized by groups while Figure 14b displays the percent difference between pre- and post-test results of each group. Similarly, Table 4 provides the descriptive data results for both pre- and post-tests obtained by each group. As can be observed, the D-D group experienced the biggest mean increase between pre- and post-test (30%); they are followed by the C-C group (22%) and the D-C group (21%), with the C-D group (19%) obtaining the smallest increase. It is interesting to also note that the D-D group had the highest post-test mean (65%), followed by the D-C group (61%), then the C-D group (57%), and finally the C-C group (53%).



(a) (b)

Figure 14. (a) Post-test results by groups; (b) Percentage difference between pre- and post-tests by groups

		N	Median	Mean	Diff. Means	Std. Dev.	Std. Error	Min	Max
C-C	Pre	10	33.33	31.25	21.63	13.07	4.13	12.50	50.00
	Post		56.08	52.88		18.31	5.79	29.17	83.33
D-D	Pre	10	34.95	35.74	29.69	15.48	4.90	16.67	66.67
	Post		66.67	65.43		11.95	3.78	41.67	83.33
D-C	Pre	10	37.50	40.00	20.83	14.86	4.70	25.00	70.83
	Post		60.42	60.83		11.49	3.63	45.83	83.33
C-D	Pre	9	36.28	38.29	18.65	8.81	2.94	29.17	54.17
	Post		54.17	56.94		17.31	5.77	41.67	91.67

Table 4. Summary of test results by groups

To complement the above descriptive data, Table 5 summarizes the results of the statistical analysis for the pre- and post-test results by groups. As can be observed, all

four treatment groups experienced significant gains in their results from pre- to post-test. One sided pair-sample *t*-tests were conducted for each group. The hypothesis tested was that participants would perform better on the post-test than on the pre-test. The results indicate a significant improvement for all four groups ($t_{C\text{-}C}(9)=4.59$; $P=.001$; $t_{D\text{-}D}(9)=7.76$; $P<.001$; $t_{D\text{-}C}(9)=6.03$; $P<.001$; $t_{C\text{-}D}(8)=4.33$; $P=.003$).

Overall test scores: paired samples *t*-test for each group

	Mean difference between pre- and post-test results	Std. deviation	*t*-statistic	Df	*p*-value
C-C	-21.633	14.904	-4.590	9	.001
D-D	-29.686	12.102	-7.757	9	.000
D-C	-20.804	10.931	-6.025	9	.000
C-D	-18.654	12.916	-4.333	8	.003

Table 5. Dependent *t*-tests for the difference between pre- and post-test scores by treatment group

Finally, to assess whether there was a significant difference between the four groups, a one-way ANOVA test was conducted on the difference between the groups' pre- and post-test (see Table 6). The results of the test show that there was no statistical significance ($F(3,35)=1.39$, $P=.263$). Similarly, Duncan's post hoc test for differences between means was conducted (see Table 7), with the results indicating no significant difference between the groups.

	Sum of Squares	Df	Mean Square	F	*p*-value
Between groups	681.419	3	227.140	1.388	.263
Within groups	5727.096	35	163.631		
Total	6408.515	38			

Table 6. Output from the one-way ANOVA test

Overall tests scores: Duncan[a,b] test for difference between means

Groups	N	Subset for alpha =.05
		1
C-C	10	21.633
D-D	10	29.686
D-C	10	20.827
C-D	9	18.654
Sig		.090

[a] Uses Harmonic Mean Sample Size = 9.730

[b] The group sizes are unequal. The harmonic mean of the group sizes is used. Type I error levels are not guaranteed.

Table 7. Duncan's test assessing the differences between means of the difference between pre- and post-test scores for each treatment group

4.1.2 Questionnaire answers

After writing the post-test, participants were requested to complete a questionnaire about the tool they used. Distilled results from this questionnaire are provided in Table 8. There appeared to be a positive correlation between participants' perception of easiness of use of a particular version, its usefulness, and preferred choice for future use. Most participants found the C-C interface to be the easiest to use (71%), the most useful (60%),

and would prefer to use it in the future if they needed to learn about 3D shapes (61%)—hence, making it the version which most participants liked (56%). In contrast, there seemed to be an inverse correlation between ease of use and the amount of thinking involved during exploration. Although very few participants (6%) did not think the C-D was easy to use, a large number (43%) thought that it would make them think hard to complete the tasks. Similarly, a relative large number of participants (36%) thought the C-D interface involved the most amount of thinking. On the other hand, very few participants (6%) said that the C-C interface involved a lot of thinking, while a large number (38%) suggested that this interface involved the least amount of thinking. In the same way, only a few participants (17%) said that the D-D interface was the easiest to use, when compared to the number of participants (about 34%) who said it would not only make them think hard about the shapes but the D-D interface also needed the most amount of thinking when completing the tasks. Finally, although a large number of participants (38%) said that the D-C interface was the hardest to use, an almost equal number (37%) said it encouraged them to visualize the shapes in their heads.

Questions	1	2	3	4	5	6	7	8	9	10
C-C	55.56	9.09	71.43	0.00	5.71	37.14	6.06	42.86	60.00	61.11
C-D	8.33	27.27	5.71	38.24	42.86	14.29	36.36	5.71	2.86	2.78
D-C	19.44	24.24	5.71	38.24	17.14	28.57	24.24	37.14	25.71	25.00
D-D	16.67	39.39	17.14	23.53	34.39	20.00	33.33	14.29	11.43	11.11

Questions:

1. Which version of the program do you like the most?
   (a) C-C, (b) C-D, (c) D-C, (d) D-D
2. Which version of the program do you like the least?
3. Which version of the program do you find the easiest to use?
4. Which version of the program do you find the hardest to use?
5. Which version of the program makes you think harder about the shapes?
6. Which version of the program needs the least amount of thinking when exploring the shapes?
7. Which version of the program needs the most amount of thinking when exploring the shapes?
8. Which version of the program encourages you to visualize and see the shapes in your head?
9. Which version of the program is more useful in helping you to explore and learn about 3D shapes?
10. Which version of the program will you use to help you study and learn about 3D shapes in the future?

Table 8. Distilled results from the questionnaire about the four versions of the tool

4.1.3 Performance and time spent on tasks

In general, all groups were able to complete the set of sixteen question-based tasks successfully using the visualization tool. Although performance on the tasks was approximately equal for all groups, they differed substantially in the time spent completing these tasks (see Table 9). As can be observed, on average, the D-C group needed only 59 minutes to complete all the tasks, the D-D group nearly 75 minutes, the C-D group about 84 minutes, and C-C almost 89 minutes. When compared to the D-C group, the D-D group spent almost 27% more time, the C-D group roughly 43%, and the C-C group nearly 51%—that is, approximately twice as much as the D-C group.

Groups	Total average time (in minutes)	Difference
D-C	59.00	+0.00 (0%)
D-D	75.00	+16.00 (27%)
C-D	83.75	+24.75 (43%)
C-C	89.25	+30.25 (51%)

Table 9. Average time spent on completing the tasks by groups

4.2. Analysis of the qualitative results

This subsection is intended to complement the above analysis by concentrating on analyzing issues that could not have been easily captured and measured using quantitative data. The data used in this analysis came mainly from transcriptions of video tape and screen capture recordings and researchers' notes.

4.2.1 C-C

As the data in Table 8 show, a large proportion of participants (60%) liked this interface the most. Participants seemed to have liked the ability to perform continuous action and receive continuous reaction. Because of this, participants felt that they had a great deal of control of the communication and information flux. When asked whether they liked this interface, participants in this group responded overwhelmingly that they did like it, especially how it allowed them to interact with and explore the 3D shapes. From their answers and comments, it appeared that the issue of control was important for this group, with strong emphasis on how they could perform continuous action. When requested to provide the reason for their liking, participants commented that "*You have full control over every movement*" with this interface, "*You manipulate it at you own rhythm and you can see the changes better*", and "*Because you can witness the transformation right before your eyes, you are in complete control of the shapes' outcome*". Similarly, other participants commented that "*I can visualize the transition of the shapes every step of the way and I can control where the shapes go by myself*", and "[The tool] *allows both direct and manual control*", giving me "*a lot of flexibility*".

Besides their liking of continuous action, participants also found the continuous and immediate reactive response to their actions to be useful and important for their exploration of the shapes. One participant said he liked this interface "*Because this version is easy to explore the shapes and shows me the resultant shapes immediately as I drag the mouse*". In the same way, other participants said that with this interface "*You can watch the shape change as you move it around*", and "*You can simultaneously drag and observe the changes*". Other related comments are: "*The transitions of the shapes are clearly shown and visible*", "*I can clearly see how they* [the shapes] *change while I move* [the Selected solid]" and "*I can see how they* [the shapes] *change smoothly and efficiently.*"

Both continuity of action and reaction appeared to have made the interface to be perceived as easier, more direct, more convenient, and more intuitive to use. For instance, participants commented that "[The C-C interface] *is simple and easy to use*" and "*It's convenient and easy to see the changes*". For one participant, this interface was the "*easiest to use and understand*", while for another participant it was "*the most efficient*

and user-friendly". And, for one other participant, it was "*intuitive* [as] *it clearly reflects/shows the transitional processes*".

4.2.2 C-D

As the data from Table 8 indicate, this form of interaction flow is the second least liked by participants, after the D-D form. When participants were asked why they did not like it, most of the reasons were about the discreteness of reaction. That is, similar to the C-C interface, participants seemed to have liked to be able to perform continuous actions, but did not like the discreteness of reactions provided in this C-D interface. In C-D interface, there was a delay between action and reaction as the Enlarged Solid would not update itself until the mouse-button had been released. Because of this delay, participants complained that they could not '*see*' the transitional changes being taking place during interaction. For example, one participant stated that "*It shows you the change but without* [showing] *the process*", and another participant commented that "*The Enlarged Shape cannot show the change clearly and, when you move it, it won't be changed. So, I can't see the change clearly*". These two participants were referring to the fact that while performing continuous actions, they would not be shown the effect on the Enlarged Solid simultaneously, but only after the action had stopped. Another participant said "*It's less visual*" because "*you can't see the constant flow of how it changes*". Similarly, other participants expressed that "*The transitions are not displayed clearly*", and "*It doesn't allow me to see clearly the transitional changes of the 3D shapes*".

A large number of participants (42%) said that this interface would have made them think hard about the shapes while completing the tasks (see Table 8). For example, one participant stated that "*It is harder to see the changes in the shape. If you don't know what you are looking for; it's very hard to find*". As such, encouraged by the flexibility of performing continuous actions, participants did more exploration—one reason why this group spent more time than the D-D and D-C groups in completing the tasks. While doing this exploration, participants often needed to imagine what the resulting shapes could be, as one participant explained "*You have to think about where you should stop to see the shape*". One other participant commented that "*The display of the changes happens too abruptly… it makes the graph harder to learn*". This increased level of difficulty might have been positive as it encouraged visualizing the resulting solids in the participants' minds first during interaction. This is supported by these two comments: "*It's choppy and doesn't flow as easily so it makes you have to visualize more*", and "*It doesn't show the* [resulting] *shape during the process. So we need to imagine it in our mind*".

4.2.3 D-D

Participants liked this form of flow the least (see Table 8). Comments from participants, however, were somewhat mixed, with some participant expressed strong liking for this interface. In general, some participants did not seem to like performing discrete actions. More importantly, they had difficulties in visualizing and understanding how a solid could morph to become another because the reactions were provided discretely. To compensate for this, participants were observed making successive and continuous mouse-clicks on adjacent, close-by locations, a dominant practice during earlier interactions with the tool. That is, it appeared that participants wanted to simulate

continuity of action, but especially, to observe the continuous transitional change of the shapes. Participants explained: "*It* [the tool] *doesn't show the transitional processes*", "*You don't understand what is going on*", and "*Sometimes is hard to tell how the polygons* [i.e., solids] *move*". Similarly, another participant said because "*It shows the least amount of transitional change, if I want to observe the trends of changing, I have to click the map lots of times*".

This practice of continuous clicking on adjacent locations on the maps became less regular and dominant as participants became more accustomed to how the tool operated. They were observed being more efficient and doing more premeditated thinking before acting. That is, participants seemed to have been able to mentally visualize the morphing of the solids after becoming more familiar with the tool and the solids. Also, once they realized the benefits and powers of this interface—e.g., the D-D interface allowed them to 'jumping' from one location to another discretely—they become more adept at finding answers for the different tasks rapidly. As the data in Table 9 indicate, participants in this group did not spend much time in completing the tasks, using only 75 minutes—just 6 minutes more than participants in the D-C group, the most efficient group. Participants found this feature of discrete jumping to be important. For example, participants expressed that the tool allowed them to go "*wherever you click. Perfect!*" Other comments were: "*It can show the result in a relatively short period*", "*It's fast and easy to see the answer*", "*Relatively speaking, it's more direct. It allows you to go quickly and directly to the place you want to go*", and "*I can see the* [resulting] *shapes quickly.*"

Many participants stated that this interface would make the think hard when solving the tasks (see Table 8). At the beginning, participants did have to think hard as they found it difficult to understand the transitional processes and to mentally connect the beginning with the final solid. However, this was beneficial because the interface "*encourages me to visualize the shape before you actually view it*" and "*is easy to use and help me remember the shapes and how they change*". As seen from the quantitative analysis, participants in the D-D group obtained the biggest improvements in terms of the difference between the pre- and post-test (see Figure 14).

4.2.4 D-C

In general, participants liked how they could interact with the D-C interface. In particular, they appreciated the continuous reaction provided in the form of animated morphing of the shapes. The animations seemed to have brought many benefits to participants' visual exploration and reasoning with the solids. For instance, because the animations would occur over a span of time and without any further input from participants, they had the time and freedom to observe and understand the information being communicated. For example, a few participants commented: "*It's moving slowly and clearly… It's easy to get more information*", "*It's very clear how the shape changes*", "*It changes slowly itself, helping me to see the whole process clearly*", and "*I can see the changing in detail*". Another participant stated: "*The transitional changes are clearly displayed and hence easy to understand.*"

Many participants stated that the D-C interface would encourage them to visualize the shapes in their minds (see Table 8). As animations could increase the visual explicitness of representations and draw attention to the important information, they seemed to have assisted participants in internalizing the properties of the solids and their transformations.

The following are three supporting statements: "[Automated] *movement help to think… to have the shapes in my head*", "*The slow changes make it easy to visualize the shapes in my head… The slow changes give me lots of information about how it changes*", and "*Because of the automated movements, the program creates a strong impression in my mind about how the shapes change.*" Also, animations had helped to create a sense of expectation of what the final shapes could be. For example, one participant stated that this interface made him think hard about the shapes "*because you wait in anticipation to see what the outcome will be*".

Participants raised two issues about this interface. One issue was about the speed of the animations. Many participants stated that it was a bit slow (e.g., "*Too slow, but somewhat acceptable*", or "*It's too slow. If I don't want to see the change I can't stop*"). The second issue was about control. Participants commented that they had little control over their actions with statements such as: "*Once you click, the transformation is irreversible. You are not in complete control*", "*When interacting or dragging, the shapes don't move. When shapes move, I cannot interact*", and "*It doesn't allow direct manual control. It is all automated*". These two related issues appeared to have contributed in encouraging participants to be stringent and efficient with their actions. As once an action was issued, participants would lose control of communication until the conclusion of the animation. The manner in which participants could explore the solids was intended for them to be more conscious about their actions and perform them after some deliberation as "*You cannot freely go back and forth. Therefore it requires thinking*". As such, issuing inefficient commands could be taxing, particularly in terms of time. As the data in Table 8 show, participants in this group spent the least amount of time in completing the tasks, about 59 minutes—almost 16 minutes fewer that the next group.

5. Discussion

Although all four groups did significantly better on the post-test than the pre-test, it is interesting to note that, among the four groups, the D-D group obtained the highest average mean score on the post-test and the biggest mean difference between pre- and post-tests (see Table 4). Similarly, the D-C group had the second highest post-test mean. Although this group achieved the third highest mean difference between the two tests, behind the D-D and C-C groups, they trailed by just less than one average point from the C-C group. It appears therefore that the two groups that could only perform discrete actions did better than the other two groups. Although the C-C interface was the most natural, intuitive, and simplest to use of the four interfaces, it is interesting to observe that the C-C group did not do well, underperforming especially the D-D group by relatively large margins. From the answers collected on the questionnaire, one plausible explanation can be extrapolated: ease and intuitiveness of use might not necessarily be conducive to deep thinking but can lead to a more automatic and shallow exploration. As the responses from the questionnaire showed (see Table 8), participants indicated that the C-C interface would not encourage them to think hard when exploring the shapes. On the contrary, participants suggested that this interface was the easiest to use and required the *least* amount of thinking. One participant went even as far as stating that the C-C interface was "*too intuitive*" to use. Ease of use might have made participants believe that the material was rather quite easy and simple, which would make them somewhat

overconfident about what and how much they were learning. This might have induced some degree of shallowness in participants' exploration of the shapes.

Ease of use might have also encouraged an inefficient and less stringent attitude towards actions performed while completing the tasks. The C-C and C-D interfaces enabled participants to undo their actions with ease. Although continuity of action seemed to have favored exploratory and discovery-based investigations, these investigations tended to be based on more opportunistic and trial-and-error strategies, as opposed to more planned and pre-meditated strategies. As the data in Table 9 show, the two groups using discrete actions were more efficient in completing the tasks, especially the D-C group using only 59 minutes on average. In contrast, the C-C group took the longest, using about 89 minutes, almost half an hour (51%) more than the D-C group. Similarly, the C-D group took almost 84 minutes, only 5 minutes fewer than the C-C group. To a great extent, both D-C and D-D groups seemed to have relied on more planned and strategic actions. This was especially the case with the D-C group, as there was time and other costs associated with each action. Once an action was issued, participants would lose control of communication and had to wait until the animation to end. The costs seemed to have promoted a highly efficient and frugal use of actions in this group.

In terms of how reactions were provided in the tool, there did not seem to be a clear relationship in terms of achievement scores in the tests (see Table 4). Nevertheless, it seemed to have affected participants' perception of the tool. Regards of how actions could be performed, participants appeared to have liked and preferred to receive continuous reaction. From their answers in the questionnaire (see Table 8), participants liked the C-C and D-C interfaces the most and thought these would be the most useful in helping them explore and learn about 3D shapes. Similarly, most participants would choose these two interfaces to help them study 3D shapes in the future. Interestingly, participants overwhelmingly stated that these two interfaces would encourage them to visualize the shapes in their heads. This might appear to contradict with their achievement scores in the tests, as, for example, the C-C group underperformed the other groups—obtaining almost 13 average points lower than the D-D group.

6. Conclusions and future work

This paper has presented a study to investigate how the factor of flow influences the cognitive processes of users while interacting with a visual tool. Such visual tools are important components of human epistemic and cognitive activities, and coupled with internal cognitive processes can form a network over which cognition is distributed. Visual tools can also serve to constrain, enhance, guide, and transform one's cognitive processes, as well as reduce strain on one's mental faculties by offloading content onto an external representation. The testbed for the study was 3D Platonic and Archimedean geometric solids. This study is based on the framework devised by Sedig & Liang (2006), who have identified a number of factors in the structuring of interaction that can affect the way users think with and reason about representations with which they interact. Flow is one of these factors. Four versions of the visualization tool were developed, based on the four forms of flow: continuous-continuous, continuous-discrete, discrete-continuous, and discrete-discrete, which refer to the cause (action) and effect (reaction) aspects of interaction.

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Five general conclusions can be derived from the present study. First, regardless of the version used, all participants benefited after interacting with the tool, as demonstrated by their achievement results between the pre- and post-test. Second, the different forms of flow did have distinct effects on participants' exploration of and reasoning with the 3D solids. Third, participants preferred to perform continuous actions and receiving continuous and immediate reactions, as they were perceived to be easier and better for their learning and exploration. Fourth, ease and intuitiveness of use might not have been conducive to deeper thinking and greater learning. And, fifth, discreteness of actions seemed to have encouraged participants to be more effective and economical in their use of actions, thereby eliciting a greater degree of reflective thinking and pre-planning before committing an action. This may have brought about improved learning.

The above conclusions have implications for the design of interactive visualization tools. First, designers do not necessarily need to strive for ease of use in these tools. The current study shows that this can be achieved without the need to include a highly complex and large quantity of content, but by judiciously making slight modifications to the way actions are performed (at the low event level) and reactions are provided—i.e., changing the feel, properties, and quality of interaction (i.e., its interactivity). And, second, designers, according to their needs and goals, can make available one form of interaction flow at different stages of exploration. For example, one can provide the C-C form to introduce learners to a tool, as they might find this form more natural and intuitive to use. Then, as the exploration process progresses, the flow of interaction can change to promote a different style of exploration and thinking. Encouraging different styles of exploration may then lead to better mastery and internalization of the content.

Interaction design can play an important role in the effectiveness and utility of visualization tools, as designers can operationalize an interaction in such a way that promotes thinking and reflection of the represented content. The actions that users perform (i.e. rotate, translate, zoom) coupled with the low-level events (i.e. mouse clicks and drags), if operationalized properly, can support different types of knowledge-oriented activities. The important thing to note is that the micro-level aspects and details of how interactions are structured are not only relevant, but are fundamental and critical to the proper design of visual tools that aim to facilitate activities such as reasoning, exploring, and sensemaking. This last point cannot be treated lightly, as many designers of educational tools put more emphasis on higher-level philosophical and pedagogical aspects of design and do not concern themselves with such low-level details. The tool presented in this study is only one narrow application of the factor of flow to interactive tools. All cognitive tools that involve interaction can benefit from the study and analysis of their forms of flow. This type of study can be applicable to tools across all domains, including education, visual analytics, information visualization, and finance, to name a few, as all these tools mediate how users carry out their mental information-based activities.

This paper concludes by inviting further research on interactivity design and the flow factor in particular. This research is just a small step and is mainly exploratory. Although the results from this research are indicative of the application and utility of conceptualizing the design of a tool based on the four forms of flow, more research is needed. Other research questions to be explored might be: What are other ways of operationalizing each form of flow? What other visual domains can the four forms of

flow be applied to? Should and how can the four forms be combined in one tool? Is there any particular groups of users—categorized based on their thinking, learning, and cognitive styles—that will benefit the most from a particular form of interaction flow? It is hoped that studies such as the one reported here encourage more research in this area.

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